Coupling Interfaces and their Impact in Field/Circuit Co-Simulation

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Co-simulation is an important approach for coupled systems in time domain analysis. Dealing with coupled systems, convergence depends on the computational order in which the parts are solved and on certain contraction properties. This paper takes a closer look at the coupling structure of field/circuit coupled problems and introduces a new approach, where we can derive information about stability and contraction directly from the network structure, without informations regarding the embedded EM device or the circuit part. When co-simulation is applied to embedded EM devices, the standard approach is to distinguish clearly between the field and circuit part. However, we show that cutting at the EM device boundary does not always works best and demonstrate that using a specific interface for coupling, co-simulation reached better properties, e.g. concerning the computational effort.

***Index Terms*—Circuit simulation, Coupling circuits, Circuit subsystems, Electromagnetic devices, Stability criteria, Convergence**

I. Introduction

IF the monolithic description of a complex system is not realisable and/or suitable software tools for the subsystems are available, then co-simulation is relevant choice. E.g. for embedded EM devices in rapidly changing electrical networks, standard time-integration is mostly inefficient, because of slow changing field quantities. Here, co-simulation allows to use different time integrators for the field and circuit part which can help to reduce the computational effort. Thereby, co-simulation tries to compute an approximation by solving multiple times the field and circuit system separately.

Our field/circuit application is a simple transformer. We consider magnetoquasistatics (MQS), where the magnetic field can be described by the curl-curl equation in terms of the magnetic vector potential, see [4], [5]. The EM device is modeled in 2D using FEMM, see [7]. The circuit equations are setup via Modified Nodal Analysis (MNA), which leads to a differential algebraic equation (DAE). Applying finite element method (FEM) the problem can be traced back to a common DAE-DAE coupling, see [3]. For coupled DAEs, convergence can only be guaranteed if certain contraction properties are fulfilled, see e.g. [1], [2].

II. Modeling and Analysis

Co-Simulation usually employs an iteration on time windows $[T_n, T_n+H]$ (with communication step size $H_n := T_{n+1}-T_n$). Let (k) denote the current and $(k-1)$ the old iterates. For DAEs with DAE-index-1, a co-simulation scheme can be encode by splitting functions $\vec{F}, \vec{G}$:

$$\begin{aligned} \dot{\vec{y}} &= \vec{f}(\vec{y},\vec{z}) \quad \leftrightarrow \quad \dot{\vec{\tilde{y}}} = \vec{F}\left(\vec{\tilde{y}}^{(k)}, \vec{\tilde{z}}^{(k)}, \vec{\tilde{y}}^{(k-1)}, \vec{\tilde{z}}^{(k-1)}\right) \\ 0 &= \vec{g}(\vec{y},\vec{z}) \qquad\quad\; 0 = \vec{G}\left(\vec{\tilde{y}}^{(k)}, \vec{\tilde{z}}^{(k)}, \vec{\tilde{y}}^{(k-1)}, \vec{\tilde{z}}^{(k-1)}\right) \end{aligned} \tag{1}$$

with differential $\vec{y}$ and algebraic $\vec{z}$ unknowns. Using partial derivatives $\vec{G}_{\vec{z}^{(k)}}$, $\vec{G}_{\vec{z}^{(k-1)}}$, the contraction condition reads [2]:

$$\alpha := \|\mathbf{G}^{-1}_{z^{(k)}}\mathbf{G}_{z^{(k-1)}}\|_2 < 1. \tag{2}$$

Therefore, from the perspective of co-simulation, one needs to design the coupling interface (if possible) such that the contraction factor α_n is as small as possible.

We expand the standard approach of source coupling by introducing an additional LR interface as given in Fig. 1. NW 1 and NW 2 signify an arbitrary network (circuit or EM device). Still the data transfer is managed by sources. Using a

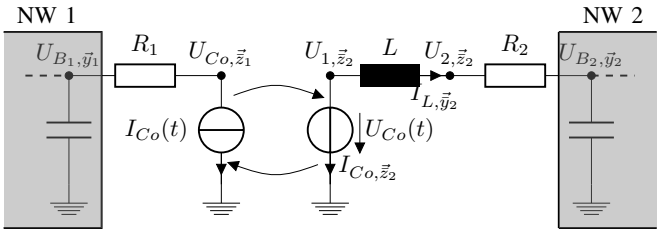

Fig. 1. Decoupling using a LR interface.

Gauß-Seidel type of iteration scheme, the splitting (extended) functions read: $\vec{y}_i$, $\vec{z}_i$ denote the variables of NW i

$$\vec{F} := \begin{bmatrix} \dot{\vec{y}}_1 \\ \dot{\vec{y}}_2 \end{bmatrix} = \begin{bmatrix} \vec{f}_1(\vec{y_1}^{(k)}, \vec{z_1}^{(k)}) + A_{f_1}\cdot G_1 \cdot U^{(k)}_{Co,\vec{z}_1} \\ \hline \vec{f}_2(\vec{y_2}^{(k)}, \vec{z_2}^{(k)}) + A_{f_2}\cdot G_2 \cdot U^{(k)}_{2,\vec{z}_2} \\ (U^{(k)}_{2,\vec{z}_2} - U^{(k)}_{1,\vec{z}_2})/L \end{bmatrix}, \tag{3}$$

$$\vec{G} := \left[\begin{array}{l} \vec{g}_1(\vec{y_1}^{(k)}, \vec{z_1}^{(k)}) \qquad\qquad + A_{g_1}\cdot G_1 \cdot U^{(k)}_{Co,\vec{z}_1} \\ (U^{(k)}_{B_1,\vec{y}_1} - U^{(k)}_{Co,\vec{z}_1})\cdot G_1 + I^{(k-1)}_{Co,\vec{z}_1}(t) \\ \hline \vec{g}_2(\vec{y_2}^{(k)}, \vec{z_2}^{(k)}) \qquad\qquad + A_{g_2}\cdot G_2 \cdot U^{(k)}_{2,\vec{z}_2} \\ I^{(k)}_{Co,\vec{z}_2} + I^{(k)}_{L,\vec{y}_2} \\ (U^{(k)}_{2,\vec{z}_2} - U^{(k)}_{B_2,\vec{y}_2})\cdot G_2 - I^{(k)}_{L,\vec{y}_2} \\ U^{(k)}_{1,\vec{z}_2} + U^{(k)}_{Co,\vec{z}_2}(t) \end{array}\right]. \tag{4}$$

The incidence matrices $\vec{A}_{f\star}, \vec{A}_{g\star}$ stamp the currents $G_1U_{Co,\vec{z}_1}, G_2U_{2,\vec{z}_2}$ into the respective branch equation. Starting from two solutions $\vec{X}$, $\tilde{\vec{X}}$ on the nth time window $[T_n, T_{n+1}]$ and performing k iterations of the co-simulation scheme, the accuracy of the solution is measured from the differences $\delta^{(k)}_{\vec{y}} := \sup_{t\in[T_n,T_{n+1}]} \|\vec{y}^{(k)}(t) - \tilde{\vec{y}}^{(k)}(t)\|_2$ (etc.). For any coupled system using the LR interface, one ends up with the (extended) recursion estimate

$$\left[\delta^{(k)}_{\vec{y}}, \delta^{(k)}_{\vec{z}}, \delta^{(k)}_{\star\vec{z}_1}, \delta^{(k)}_{\star\vec{z}_2}\right]^T \leq \boldsymbol{K_e}\cdot\left[\delta^{(k-1)}_{\vec{y}}, \delta^{(k-1)}_{\vec{z}}, \delta^{(k-1)}_{\star\vec{z}_1}, \delta^{(k-1)}_{\star\vec{z}_2}\right]^T$$

with $\delta_{\vec{y}}^{(k)} = (\delta_{\vec{y}_1}^{(k)}, \delta_{\vec{y}_2}^{(k)})^T$, $\delta_{\vec{z}}^{(k)} = (\delta_{\vec{z}_1}^{(k)}, \delta_{\vec{z}_2}^{(k)})^T$, $\delta_{\star_{\vec{z}_1}} = \delta U_{Co,\vec{z}_1}$, $\delta_{\star_{\vec{z}_2}} = (\delta U_{1,\vec{z}_2}, \delta U_{2,\vec{z}_2}, \delta I_{Co,\vec{z}_2})^T$ and (extended) recursion matrix $\boldsymbol{K}_e$. And it holds $\rho(\boldsymbol{K}_e) = H$ for any NW1 and NW2 (both of index-1). Thus convergence with order $\mathcal{O}(H)$ is guaranteed for window size $H < H_{\max}$.

III. Numerical Test

Our example is a transformer typically used in low frequency applications, connected by a wire to a voltage source, see Figs. 2. The 2D transformer model is given by FEMM using finite element method (FEM) for space discretisation, [6]. As resistances of the coil windings we use $R_{M,1} = 0.449\ \Omega$, $R_{M,2} = 0.062\ \Omega$, see [7]. This transformer has 260 turns on the primary and 90 turns on the secondary side with no-load. After semi-discretization, both are index-1 systems. Thus the splitting scheme (3),(4) can be applied. As reference,

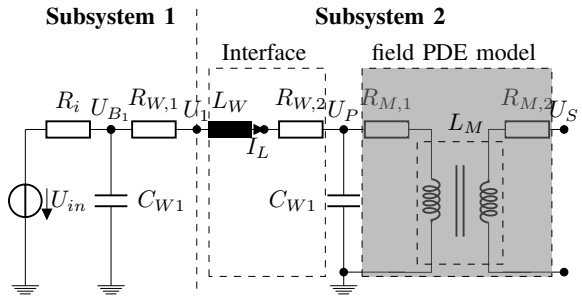

Fig. 2. Field/circuit coupling using LR interface. Settings: $R_{W,1} = R_{W,2} = 1\text{k}\Omega$, $C_{W,1} = C_{W,2} = 1\text{nF}$, $L_W = 1\text{nH}$, $R_i = 10\Omega$, and supply voltage $U_{\text{in}}(t) = 170\text{V} \cdot \cos(\omega t)$ with $\omega = 2\pi \cdot 60$ Hz. Replacement: $U_{B_2} = U_P$.

we compute a solution of the monolithic system, i.e., without co-simulation (most inefficent).

To investigate co-simulation, we studied contraction over the whole simulation time $[0, t_{\text{sim}}]$ using a fixed window size for both coupling types, i.e., classical field/circuit- and LR coupling. In both setting, we employ constant extrapolation for the coupling variables in the first iteration of each time window.

For all our investigations, we inspect the error in the primary and secondary voltages of the transformer. Fig. 3 (left) shows the contraction behaviour with LR interface and standard field/circuit decoupling for $t_{\text{sim}} = 0.02$ s and $H = 2.5$ ms (largest possible window size for the classical approach). Thus both schemes are stable and converge. That is, LR coupling converges much faster than the classical approach before both schemes are bounded by the time integrator accuracy. Secondly, we measured the convergence order (Fig. 3, right) on $[t_0, t_0 + H]$ with $t_0 = 0.001$ s. Thus, for both approaches we get order $\mathcal{O}(H)$, which fits to the preceding theory. Using LR coupling: Fig. 4 (left) shows that for increasing number of iterations, the solution of the primary and secondary voltage converges to the reference solution (solid lines). Fig. 4 (right) illustrates the error in the primary and secondary voltage for a much enlarged window size $H = 0.02$ s after $k = 15$ iterations. For each point of time the error is bounded by the time integrator accuracy. Thus, using LR coupling enables to enlarge the time window up to approximately two periods (factor

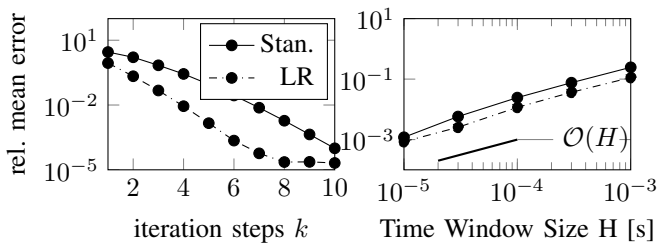

Fig. 3. (Left) Error in primary/secondary voltages in dependence of the iteration steps k in $[0, 0.02]$ s, using window size $H = 2.5$ ms for both approaches, i.e. standard- and LR coupling. (Right) Convergence of the field/circuit co-simulation model for different time window sizes H with one iteration per time window after $t_0 = 0.001$ ms with and without LR coupling.

8). Using much larger window size, a careful consideration regarding the interplay of window size and number of iterations is needed. Therefore, to benefit from it, an optimal ratio of

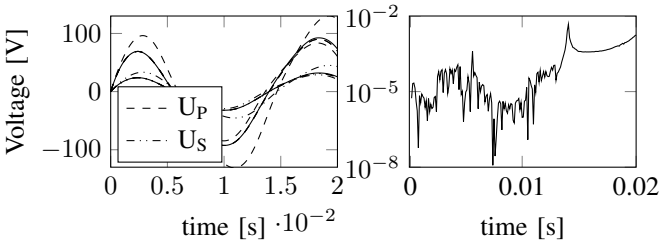

Fig. 4. (Left) Primary/Secondary voltage for $k = 1, 5, 10, 15$ iterations (LR interface). The solid lines denotes the respective reference solution. (Right) Relative error (y-axis) in primary/secondary voltage on the time window $[0, H]$ with $H = 0.02$ s performing $k = 15$ iterations.

iterations and window size needs to be investigated.

IV. Conclusion

We demonstrated that stability and contraction in field/circuit coupled problems can be derived directly from the network structure. Applying the LR structure as coupling method, co-simulation works well for each computational order and for each choice of electrical networks (provided that index-1 holds for all subsystem). Furthermore, we showed that LR coupling enables to enlarge the window sizes.

Nevertheless, the design of the coupling interface needs further attention. In the future, we also aim at including uncertainty into the convergence analysis.

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